Section 2.7	*Graphs of Rational Functions*
Objective	Given a rational function students will be able to sketch its graph. *Study Problems* *Page 204 #3-29, eoo, 77-85 odd*

Quiz Wednesday

Due Tuesday

Assessment Friday

Assessment corrections Due 10/27

Warm up

1 Write a rational function with horizontal asymptote at 3 and vertical asymptote at -2.

$y = \frac{3x}{x+2}$ or $y = \frac{3x+2}{x+2}$

2 Write a rational function with a horizontal asymptote and 2 vertical asymptotes.

$y = \frac{x+1}{x^2-4x+3}$ ✓ $y = \frac{x-1}{x^2+4x-3}$

Guidelines for Graphing Rational Functions

Let $f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials with no common factors.

1. Find and plot the y-intercept (if any) by evaluating $f(0)$.
2. Set the numerator equal to zero and solve the equation $N(x) = 0$. The real solutions represent the x-intercepts of the graph. Plot these intercepts.
3. Set the denominator equal to zero and solve the equation $D(x) = 0$. The real solutions represent the vertical asymptotes. Sketch these asymptotes using dashed vertical lines.
4. Find and sketch the horizontal asymptote of the graph using a dashed horizontal line.
5. Plot at least one point between and one point beyond each x-intercept and vertical asymptote.
6. Use smooth curves to complete the graph between and beyond the vertical asymptotes.

Example

$$k(x) = \frac{x^2 - 2x - 3}{x+2}$$

a) find the domain

b) Identify the horizontal asymptote

c) Identify the vertical asymptote

d) x and y intercepts

e) Sketch its graph

Justify your response

use a graphing calculator to verify your results

a) $x+2 \neq 0$ $\quad x \neq -2$

$\{x \in \mathbb{R} \mid x \neq -2\}$

b) None

c) $x = -2$

d) x-int $\quad y=0$

$0 = x^2 - 2x - 3$

$(x-3)(x+1) = 0$

$(3,0) \quad (-1,0)$

y-int

$x = 0$

$y = -\frac{3}{2}$

$(0, -3/2)$

e) Slant Asy

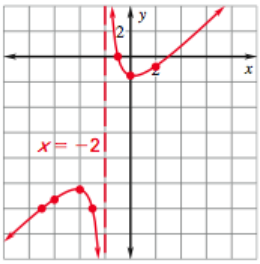
$$\begin{array}{r|rrr} x+2 & x^2 & -2x & -3 \\ -2 & 1 & -2 & -3 \\ & & -2 & 8 \\ \hline & 1 & -4 & 5 \end{array}$$

$y = x - 4$ $\quad$ Slant Asy.

Example 3 ***Graph a rational function (m > n)***

Graph $y = \dfrac{x^2 - 2x - 3}{x + 2}$.

The numerator factors as $(x - 3)(x + 1)$, so the x-intercepts are 3 and -1. The zero of the denominator $x + 2$ is -2, so the vertical asymptote is $x = -2$. The degree of the numerator, 2, is greater than the degree of the denominator, 1, so the graph has no horizontal asymptote. The graph has the same end behavior as the graph of $y = x^{2-1} = x$.

Plot points on each side of the vertical asymptote.

	x	y
To the left of $x = -2$	-7	-12
	-6	-11.3
	-4	-10.5
	-3	-12
To the right of $x = -2$	-1	0
	0	-1.5
	2	-0.8