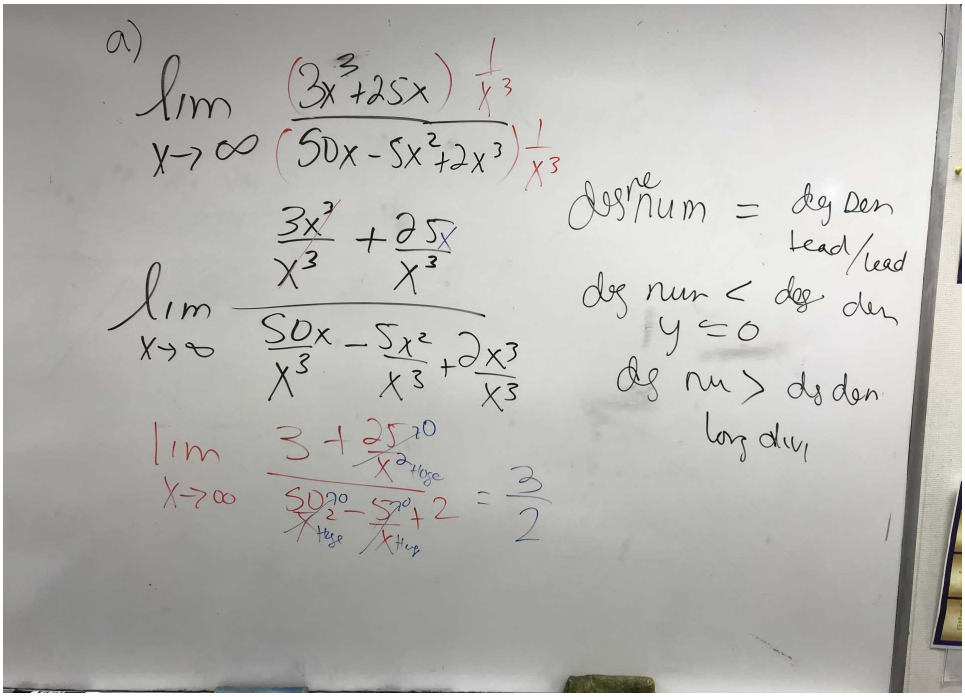
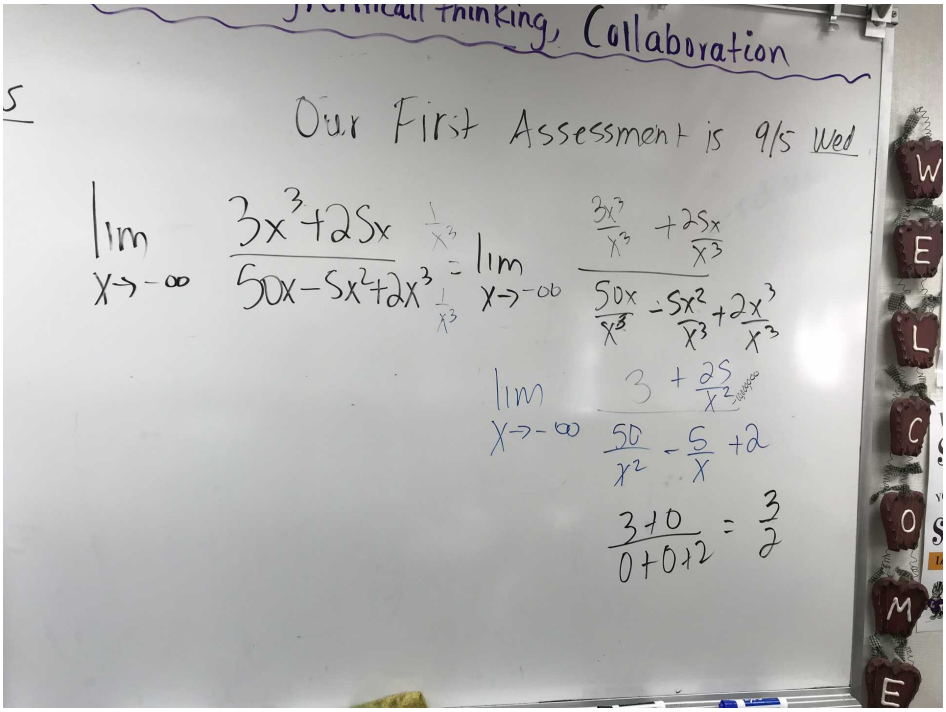
Example

Show that f(x) has a horizontal asymptote at y= 1.5

$$a.\ f(x)=\frac{3x^3+25x}{50x-5x^2+2x^3}$$





Example

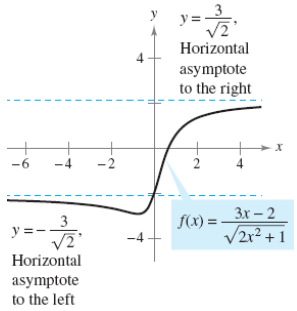
Show that f(x) has a horizontal asymptote at y= 0

$$b.\ f(x)=\frac{x^3+2x}{5x+6x^2-7x^4}$$

Example

Find the limit:

a. $\lim_{x \to \infty} \frac{3x+2}{\sqrt{2x^2+1}}$



For $x > 0$, you can write $x = \sqrt{x^2}$. So, dividing both the numerator and the denominator by x produces

$$\frac{3x-2}{\sqrt{2x^2+1}} = \frac{\frac{3x-2}{x}}{\frac{\sqrt{2x^2+1}}{\sqrt{x^2}}} = \frac{3-\frac{2}{x}}{\sqrt{\frac{2x^2+1}{x^2}}} = \frac{3-\frac{2}{x}}{\sqrt{2+\frac{1}{x^2}}}$$

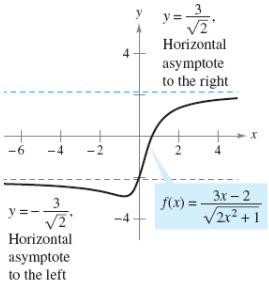
and you can take the limit as follows.

$$\lim_{x\to\infty} \frac{3x-2}{\sqrt{2x^2+1}} = \lim_{x\to\infty} \frac{3-\frac{2}{x}}{\sqrt{2+\frac{1}{x^2}}} = \frac{3-0}{\sqrt{2+0}} = \frac{3}{\sqrt{2}}$$

Example

Find the limit:

b. $\lim_{x \to -\infty} \frac{3x+2}{\sqrt{2x^2+1}}$



b. For $x < 0$, you can write $x = -\sqrt{x^2}$. So, dividing both the numerator and the denominator by x produces

$$\frac{3x-2}{\sqrt{2x^2+1}} = \frac{\frac{3x-2}{x}}{\frac{\sqrt{2x^2+1}}{-\sqrt{x^2}}} = \frac{3-\frac{2}{x}}{-\sqrt{\frac{2x^2+1}{x^2}}} = \frac{3-\frac{2}{x}}{-\sqrt{2+\frac{1}{x^2}}}$$

and you can take the limit as follows.

$$\lim_{x\to-\infty} \frac{3x-2}{\sqrt{2x^2+1}} = \lim_{x\to-\infty} \frac{3-\frac{2}{x}}{-\sqrt{2+\frac{1}{x^2}}} = \frac{3-0}{-\sqrt{2+0}} = -\frac{3}{\sqrt{2}}$$

The graph of $f(x) = (3x-2)/\sqrt{2x^2+1}$ is shown in Figure 3.38. ■