# 4.1 Antiderivatives and Indefinite Integration

Objective: Given the rules to find antiderivatives students will be able to find the integral of a given function.

## Example

Find the indefinite integral.

a) $\int 4x^4\,dx =$

b) $\int \frac{3}{x^5}\,dx$

c) $\int \sec^2 x\,dx$

a) $\frac{4x^5}{5} + C$

b) $\int 3x^{-5}dx = \frac{3x^{-4}}{-4} + C$

c) $\tan x + C$

## Example

Find the indefinite integral.

a) $\int \frac{x-6}{\sqrt{x}}\,dx =$

b) $\int \frac{x^2+2x-3}{x^4}\,dx$

c) $\int y^3\sqrt[5]{y^3}\,dy$

a) $\int x^{-1/2}(x-6)\,dx$

$\int x^{1/2} - 6x^{-1/2}\,dx$

$\frac{2}{3}x^{3/2} - 12x^{1/2} + C$

b) $\int x^{-4}(x^2+2x-3)\,dx$

$\int x^{-2} + 2x^{-3} - 3x^{-4}\,dx$

$-1x^{-1} - 1x^{-2} + x^{-3} + C$

c) $\int y^3 \cdot y^{3/5}\,dx$

$\int y^{18/5}\,dy$

$\frac{5}{23}y^{23/5} + C$

$$\frac{d}{dx}(\ln u) = \frac{u'}{u}$$

$$\int \frac{6x^5}{x^6+4}\,dx$$

$$\ln(x^6+4) + C$$

## Example

Find the indefinite integral.

a) $\int \frac{\sin x}{\cos x}\,dx$

b) $\int \frac{\sin x}{\cos^2 x}\,dx$

$\int \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right) dx$

$\int \sec x \tan x\, dx$

$\sec x + C$

a) $= \int \tan x\, dx$

$= \int \frac{-\sin x}{\cos x}\,dx$

$= \ln|\cos x| + C$

b) $\int \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}\,dx$

$\int \sec x \tan x\, dx$

$\boxed{\sec x + C}$

## Example

Find the indefinite integral.

a) $\int \sec y(\tan y - \sec y)\, dy$

b) $\int \frac{\tan x}{\sec x}\,dx$

c) $\int (3x^2 - \csc^2 x)\, dx$

a) $= \int \sec y \tan y - \sec^2 y\, dy$

$\sec y - \tan y + C$

b) $= \int \frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}\, dx$

$\int \sin x\, dx = -\cos x + C$

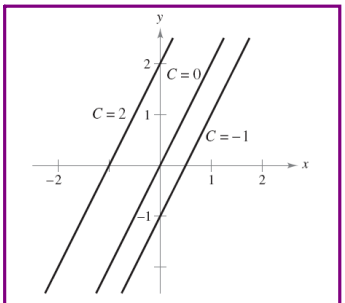
c) $x^3 + \cot x + C$

Sometimes the problems will ask you to find the C at which the function needs to pass through.

Remember
y'=2

*General Solution*

This has many solutions $\int 2\,dx = 2x + C$



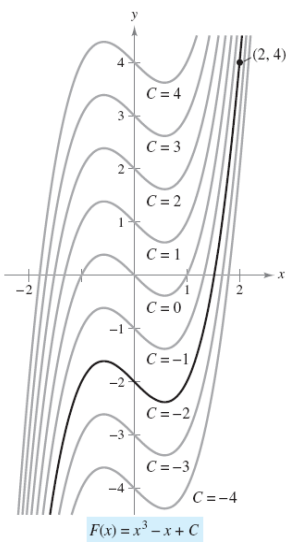
Functions of the form $y = 2x + C$

Now you will be given an ordered pair to find the C.
For example, your particular solution goes through (0,2)

$y = 2x + C$

$2 = 2(0) + C$ $\quad C = 2$

$y = 2x + 2$




Find a general solution and then find a particular solution for (2,4).

$$\frac{dy}{dx} = 3x^2 - 1.$$

$$y = \int (3x^2 - 1)\,dx = x^3 - x + C$$ General solution

$$F(x) = x^3 - x + C$$ General solution

$$F(2) = 4$$ Initial condition

By using the initial condition in the general solution, you can determine that $F(2) = 8 - 2 + C = 4$, which implies that $C = -2$. So, you obtain

$$F(x) = x^3 - x - 2.$$ Particular solution

## EXAMPLE 7 Finding a Particular Solution

Find the general solution of

$$F'(x) = \frac{1}{x^2}, \quad x > 0$$

and find the particular solution that satisfies the initial condition $F(1) = 0$.

General Solution

$y = -\frac{1}{x} + C$

Particular Soln.

$y = -\frac{1}{x} + 1$

## Example

Find a particular solution for the integrand.

a) $\int \frac{2}{\sqrt{x}}\,dx$, (4,10)

b) $\int (t^2 + 1)^2\,dt$, (0,1)

c) $\int \frac{x^3 + 3}{x^2}\,dx$, (1,2)

d) $\int \sqrt[3]{x}(x - 4)\,dx$, (0,3)

a) $\int 2x^{-1/2}\,dx$

$2x^{1/2} \cdot 2 + C$

$y = 4x^{1/2} + C$

$10 = 4(4)^{1/2} + C$

$10 = 8 + C$

$C = 2$

$\boxed{y = 4x^{1/2} + 2}$

b) $\int t^4 + 2t^2 + 1\,dt$

$y = \frac{t^5}{5} + \frac{2t^3}{3} + t + C$

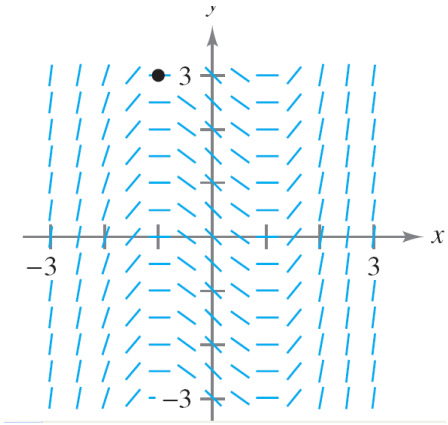
$C = 1$

$y = \frac{t^5}{5} + \frac{2t^3}{3} + t + 1$

## Example

Find a particular solution for the integrand.

$$\frac{dy}{dx} = x^2 - 1, (-1, 3)$$




$$\int x^2 - 1 \, dx$$

$$y = \frac{x^3}{3} - x + C$$

$$3 = \frac{(-1)^3}{3} - (-1) + C$$

$$C = \frac{7}{3}$$

$$y = \frac{x^3}{3} - x + \frac{7}{3}$$