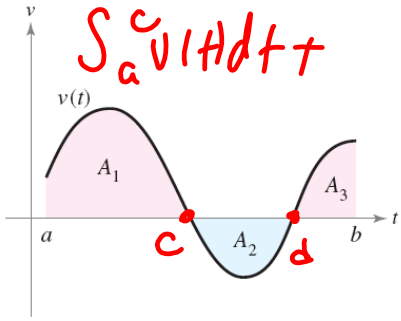
When calculating the *total* distance traveled by the particle, you must consider the intervals where $v(t) \le 0$ and the intervals where $v(t) \ge 0$. When $v(t) \le 0$, the particle moves to the left, and when $v(t) \ge 0$, the particle moves to the right. To calculate the total distance traveled, integrate the absolute value of velocity $|v(t)|$. So, the displacement of a particle and the total distance traveled by a particle over $[a, b]$ can be written as

Displacement on $[a, b]$ $= \int_a^b v(t)\,dt = A_1 - A_2 + A_3$

Total distance traveled on $[a, b]$ $= \int_a^b |v(t)|\,dt = A_1 + A_2 + A_3$



A_1, A_2, and A_3 are the areas of the shaded regions.

Calculator

Quiz tom.
4.4 & 4.5
FR

EXAMPLE 10 Solving a Particle Motion Problem

A particle is moving along a line so that its velocity is $v(t) = t^3 - 10t^2 + 29t - 20$ feet per second at time t.

a. What is the displacement of the particle on the time interval $1 \le t \le 5$?

b. What is the total distance traveled by the particle on the time interval $1 \le t \le 5$?

a) $\int_1^5 v(t)\,dt = 10.\overline{6}$ ft

b) $\int_1^5 |v(t)|\,dt = 11.8\overline{3}$ ft

Example

Calculator

$$v(t) = 3e^{\left(-t/2\right)} \sin(2t)$$

A particle travels along a straight line with a velocity of the function above, in meters per second. What is the total distance in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

zero 1.57

2.261

2.261 m

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Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

$y_1 =$ NINT

(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$
$v(t)$ changes sign from negative to positive at time $t = 0.536033$.
$v(t)$ changes sign from positive to negative at time $t = 3.317756$.

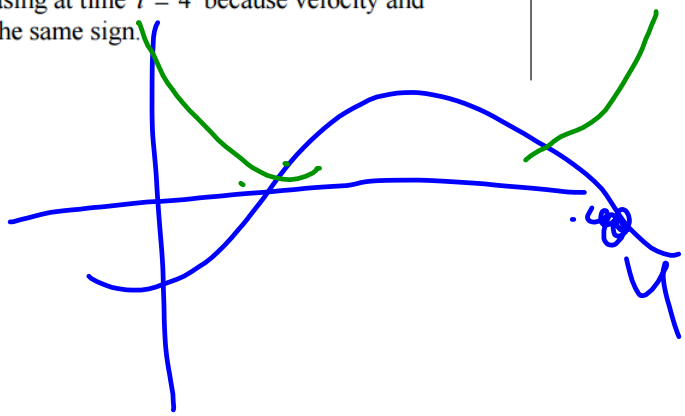
Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

3 : { 1 : considers $v(t) = 0$; 2 : answers with justification }

(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

2 : conclusion with reason



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Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

2 : { 1 : integrand; 1 : answer }

(b) $R(3) - D(3) = -0.313632 < 0$
Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

2 : { 1 : considers $R(3)$ and $D(3)$; 1 : answer and reason }

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is $30 + \int_0^t [R(x) - D(x)]\,dx$.

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

3 : { 1 : considers $R(t) - D(t) = 0$; 1 : answer; 1 : justification }

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

2 : { 1 : integral; 1 : equation }

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Question 2

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

(a) What was the distance between the road and the edge of the water at the end of the storm?

(b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

(c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

(d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

(a) $35 + \int_0^5 f(t)\,dt = 26.494$ or 26.495 meters

2 : { 1 : integral; 1 : answer }

(b) Four hours after the storm began, the rate of change of the distance between the road and the edge of the water is increasing at a rate of 1.007 meters / hours^2.

2 : { 1 : interpretation of $f'(4$; 1 : units }

(c) $f'(t) = 0$ when $t = 0.66187$ and $t = 2.84038$
The minimum of f for $0 \le t \le 5$ may occur at 0, 0.66187, 2.84038, or 5.

$f(0) = -2$
$f(0.66187) = -1.39760$
$f(2.84038) = -2.26963$
$f(5) = -0.48027$

The distance between the road and the edge of the water was decreasing most rapidly at time $t = 2.840$ hours after the storm began.

3 : { 1 : considers $f'(t) = 0$; 1 : answer; 1 : justification }

(d) $-\int_0^5 f(t)\,dt = \int_0^x g(p)\,dp$

2 : { 1 : integral of g; 1 : answer }