Section 5.5	Double Angle Formulas
Objective:	Given equations students will use double angle formulas to rewrite and evaluate trigonometric functions.

Study Problems

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Theorem Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta \\ &= 2\cos^2\theta - 1\end{aligned}$$

Theorem Double-Angle Formula for Tangent

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Double Angle Formula

$$\sin 2x = 2\sin x \cos x$$

$$\sin 4x =$$

$$\sin 6x =$$

$$\sin bx =$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 4x =$$

$$\cos 6x =$$

$$\cos bx =$$

Example

Find the exact value of

$\boxed{\text{if } \sin\theta = \frac{5}{13},}$ $\frac{\pi}{2} < \theta < \pi$

a) $\sin 2\theta =$

b) $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{5}{13}\right)^2 = 1 - 2\left(\frac{25}{169}\right) = 1 - \frac{50}{169}$

$\boxed{\frac{119}{169}} = \frac{169}{169} - \frac{50}{169}$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

or

$1 - 2\sin^2\theta$

or

$2\cos^2\theta - 1$

or

$2\cos^2\theta - 1$

$2\left(-\frac{12}{13}\right)^2 - 1 = 2\left(\frac{144}{169}\right) - 1$

$\frac{288}{169} - \frac{169}{169} = \boxed{\frac{119}{169}}$

Example

Find the exact value of

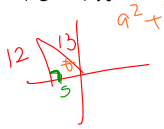
$$\text{if } \sin\theta = \frac{12}{13}, \frac{\pi}{2} < \theta < \pi$$

a) sin2θ

b) cos2θ

c) tan2θ

$a^2+b^2=c^2$



a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$

$\boxed{\frac{-120}{169}}$

b) $1 - 2\sin^2\theta$

$1 - 2\left(\frac{12}{13}\right)^2 = 1 - \frac{288}{169}$

$\boxed{-\frac{119}{169}}$

c) $\dfrac{\frac{-120}{\cancel{169}}}{\frac{-119}{\cancel{169}}} = \frac{-120}{-119}$

$\frac{120}{119}$

Example

Find all values of x

$$\sin 2\theta \sin\theta = \cos\theta$$

$(2xy)x$

$2x^2y$

$(x-1)(x+2)=0$

$x-1=0 \quad x+2=0$

$(2\sin\theta\cos\theta)\sin\theta = \cos\theta$

$2\sin^2\theta\cos\theta = \cos\theta$

$2\sin^2\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin^2\theta - 1) = 0$

$\cos\theta = 0$ or $2\sin^2\theta - 1 = 0$

$\theta = \frac{\pi}{2} + \pi k$

k is an integer

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\sqrt{\frac{1}{2}}$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$

the 1st or 2.

① $\theta = \frac{\pi}{4} + 2\pi k$ ② $\theta = \frac{3}{4}\pi + 2\pi k$

③ $\theta = \frac{5}{4}\pi + 2\pi k$ ④ $\theta = \frac{7}{4}\pi + 2\pi k$

where k is an int.

Example

Find all values of x

$$\sin 2\theta - \cos\theta = 0$$

$\sin 2x - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = 1/2$

~~$x = \pi/2, 3\pi/2$ or $x = \pi/6$ or $5\pi/6$~~

$\frac{\pi}{2} + \pi K$

$\frac{\pi}{6} + 2\pi K$

$\frac{5}{6}\pi + 2\pi K$

Example

Verify the identity.

$$\csc 2\theta = \frac{\csc\theta}{2\cos\theta}$$

$\csc 2\theta = \frac{1}{\sin 2\theta}$

$= \frac{1}{2\sin\theta\cos\theta}$

$\frac{1}{2} \cdot \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$

$\frac{\csc\theta}{2\cos\theta}$ ✓