

Unit 3 Mod 6.5 Part 2 "Synthetic Division

Divide each of the polynomials using long division. Remember to place zeros for the missing terms in the polynomial.

$$\begin{array}{r}
 12x^3 + 20x^2 + 11x + 2 \\
 3x+2 \overline{) 12x^3 + 20x^2 + 11x + 2} \\
 \underline{12x^3 + 8x^2} \\
 12x^2 + 11x \\
 \underline{12x^2 + 8x} \\
 3x + 2 \\
 \underline{3x + 2} \\
 0
 \end{array}$$

$4x^2 + 4x + 1$

$$\begin{array}{r}
 12x^3 + 20x^2 + 11x + 2 \\
 2x+1 \overline{) 12x^3 + 20x^2 + 11x + 2} \\
 \underline{12x^3 + 6x^2} \\
 14x^2 + 11x \\
 \underline{14x^2 + 7x} \\
 4x + 2 \\
 \underline{4x + 2} \\
 0
 \end{array}$$

$6x^2 + 7x + 2$

Given a polynomial $p(x)$, use synthetic division to divide by $x - a$ and obtain the quotient and the (nonzero) remainder. Write the result in the form $p(x) = (x - a)(\text{quotient}) + p(a)$. You may wish to carry out a check.

9. $(7x^3 - 4x^2 - 400x - 100) \div (x - 8)$

$$\begin{array}{r|rrrrr}
 8 & 7 & -4 & -400 & -100 & \\
 & & 56 & 416 & 128 & \\
 \hline
 & 7 & 52 & 16 & 28 &
 \end{array}$$

$7x^2 + 52x + 16 + \frac{28}{x-8}$

check:

$$\begin{aligned}
 &(x - 8)(7x^2 + 52x + 16) + 28 \\
 &= 7x^3 - 56x^2 + 52x^2 - 416x + 16x - 128 + 28 \\
 &= 7x^3 - 4x^2 - 400x - 100 \leftarrow \text{dividend}
 \end{aligned}$$

10. $(8x^4 - 28.5x^2 - 9x + 10) \div (x + 0.25)$

$$\begin{array}{r|rrrrrr}
 -0.25 & 8 & 0 & -28.5 & -9 & 10 & \\
 & & -2 & 0.5 & 7 & -0.5 & \\
 \hline
 & 8 & -2 & -28 & -2 & 9.5 &
 \end{array}$$

$8x^3 - 2x^2 - 28x - 2 + \frac{9.5}{x+0.25}$

check:

$$\begin{aligned}
 &(x + 0.25)(8x^3 - 2x^2 - 28x - 2) + 9.5 \\
 &= 8x^4 + 2x^3 - 2x^3 - 0.5x^2 - 28x^2 - 7x - 2x \\
 &\quad - 0.5 + 9.5 \\
 &= 8x^4 - 28.5x^2 - 9x + 10 \leftarrow \text{dividend}
 \end{aligned}$$

11. $(2.5x^3 + 6x^2 - 5.5x - 10) \div (x + 1)$

$$\begin{array}{r|rrrrr}
 -1 & 2.5 & 6 & -5.5 & -10 & \\
 & & -2.5 & -3.5 & 9 & \\
 \hline
 & 2.5 & 3.5 & -9 & -1 &
 \end{array}$$

$2.5x^2 + 3.5x - 9 + \frac{-1}{x+1}$

check:

$$\begin{aligned}
 &(x + 1)(2.5x^2 + 3.5x - 9) - 1 \\
 &= 2.5x^3 + 2.5x^2 + 3.5x^2 + 3.5x - 9x - 9 - 1 \\
 &= 2.5x^3 + 6x^2 - 5.5x - 10 \leftarrow \text{dividend}
 \end{aligned}$$

Given $p(x)$, find $p(-3)$ by using synthetic substitution.

1. $p(x) = 8x^3 + 7x^2 + 2x + 4$

$$\begin{array}{r|rrrr}
 -3 & 8 & 7 & 2 & 4 \\
 & & -24 & 51 & -159 \\
 \hline
 & 8 & -17 & 53 & -155
 \end{array}$$

$p(-3) = -155$
or
 $(-3, -155)$

2. $p(x) = x^3 + 6x^2 + 7x - 25$

$$\begin{array}{r|rrrr}
 -3 & 1 & 6 & 7 & -25 \\
 & & -3 & -9 & 6 \\
 \hline
 & 1 & 3 & -2 & -19
 \end{array}$$

$p(-3) = -19$
or
 $(-3, -19)$